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THE ASYMMETRY OF THE IMAGINATION[1]

IT appears that human minds, at least some of them, are blessed with a variety of lights. There is the light of ocular perception, commonly objectified, and usually identified with solar radiance or with that of electricity or that of a burning tallow candle. There is the light of the imagination in which, with or without the eyes being closed, one may behold things that may or may not correspond to things previously beheld in perceptional light and that, following the psychological nomenclature of the ancient atomists, are often unhappily called "images," as if they were given-off masks or copies of objects dwelling and revealed in the light of perception. If the metaphor may be allowed, the rays of imagination's light are not always straight. They may bend, curve, and converge so as to present a view of something around the corner, or of an aspect turned away from the beholder as the opposite phase of a sphere. Every one may know that in the light of imagination things are disclosed whose perceptional correspondents have not been seen and may not exist. Indeed, the geometrician finds his occupation to consist largely in the beholding and contemplation of configurations whose conceptual correspondents may have been previously indicated by analysis as distinguished from spatial intuition, or which may, after their own manner, have just "dawned upon his imagination," without their ever having been clad in ether-reflecting garb or touched by a solar ray. Then there is the light of thought—the light of analysis, whose objects, like number, for example, do not inhabit space—very different from the other two kinds of light, illuminating the way where the light of perception fails, penetrating infinitely beyond the utmost reaches of the light of imagination, knowing no bounds save the barriers and limitations of logic; the beautiful, gray, ubiquitous light it is in which one beholds, suspended in their nakedness and purity, the intellectual structures known as theory, woven of concepts and relationships—the ganglia and netted neural fibers of being—witnessing to the rationality of the human spirit and serving it as types and standards for the criticism and estimate of worlds. That there is a yet higher and finer kind of light, a light peculiar to the higher emotions, a radiance and glory of things not revealed in the lights of ocular perception or imagination or thought—I own to having something more than a mild suspicion that there is such a radiance and glory, but I shall not pursue the inquiry in this connection. Of the varieties of light mentioned, it is that of imagination and that of thought, or rather it is the powers or faculties, imagination and thought, which reveal those lights and the things that dwell therein, it is these faculties that I wish to speak of here.

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[1] Paper read before the tenth annual meeting of the American Philosophical Association at Princeton, December 28, 1910.

Some years ago, when lecturing on a hypercomplex of geometric elements in space of fourfold dimensionality in points, I was struck by a strange phenomenon in the field of the imagination. The hypercomplex—call it H—was represented by an equation, as $F=0$. Now four-space is exactly as rich in lines as it is in planes, containing a sixfold infinity of elements of either kind. The ten variables in F were connected by just enough identities to leave the variables equivalent to six independents, and thus to render them available as line coordinates or as plane coordinates in four-space. Accordingly, as the equation, $F=0$, imposed one additional condition on the variables, the H represented by the equation was a configuration composed of a fivefold infinitude of lines, if the variables were regarded as line coordinates, or of a fivefold infinitude of planes if the variables were regarded as plane coordinates: a rather numerous array of elements when taken absolutely, but meager enough in comparison with the total ensemble of lines or planes contained in four-space. The question was, supposing H to be composed of lines, how were the lines distributed or disposed throughout four-space? What sort of configuration was this H? What did it look like? What was the character of its make-up and constitution? And, supposing H to be composed of planes, the same question was put concerning them. How were the planes distributed? How disposed and related? In a word, H being composed of lines or of planes, what were the properties of H? To answer, it was necessary to dissect H, to cut it, so to speak, with simpler configurations, and to examine the traces or intersections; that is, to examine the configurations composed of the elements common to H and the cutting configurations. If H as line configuration were cut by certain line configurations C_1, C_2, $\cdots$, the intersections were certain other line configurations γ_1, γ_2, $\cdots$. If H as plane configuration were cut by plane configurations C', C'', $\cdots$, the intersections were certain other plane configurations γ', γ'', $\cdots$. When C_1, C_2, $\cdots$ were respectively the same logically, *i. e.*, in thought, as C', C'', $\cdots$, then γ_1, γ_2, $\cdots$ were for thought, *i. e.*, logically, the same respectively as γ', γ'', $\cdots$. In this fact there was nothing to surprise, it was evident *a priori*, for H as line configuration and H as plane configuration are, by virtue of their identical algebraic definition, $F = 0$, one and the same in logical make-up, in conceptual constitution, in their character as thought structures. The surprise came when it was observed—and this is the unexpected phenomenon alluded to above—that, albeit the γ-configurations are one-to-one logically identical with the γ'-configurations, the former do not disclose themselves in the light of imagination equally with their correspondents among the latter. A given γ-configuration may disclose itself wholly, partially, or not at all without the same being true, or true in the same measure, of the corresponding γ'-configuration. So that, whilst thought pursued a straightforward course in the domain under consideration, imagination was constrained by its own limitations to a zigzag course. Whilst thought spread its characteristic radiance equally everywhere throughout the fourfold domain, the brighter light of the imagination shone but here, there, and yonder, in patches and spots, like the patines of golden light that of a clear summer day one may see on leaf and ground and tree-trunk in the depths of a thick forest.

The phenomenon ought, it seemed, to be of interest to students of psychology. Its bearings upon other matters may or may not be important. Time, if the matter be followed up, may determine that. The immediate task is to bring the phenomenon into clearer view, to display it, if possible, in a field less remote and less difficult of access than that in which it was first observed, and thus to render it observable and unmistakable to such as may not be prepared to pursue far the ways of geometric interpretation. This task can be performed as follows by the help of considerations demanding but a very slight knowledge of algebraic methods applied to the study of space. It will be observed that the procedure exemplifies the possibility of employing analytic geometry as an instrument for investigating some aspects of the nature and behavior of the human mind. I have no doubt the time will come when mathematical methods will be recognized as an indispensable part of the proper equipment of the philosopher and the psychologist, and when, accordingly, in our colleges and universities there will be regularly offered mathematical courses specially designed for such students. That the applications of mathematics are destined to be confined to fields now conventionally described as the fields of natural science is a belief having its ground in a rather prosaic infidelity respecting the similitudes that penetrate and ally all the forms of spiritual activity.

In the title of this paper imagination is described as asymmetric.

On the other hand, it will be contended that thought, at least in the domain under consideration, is symmetric. What precisely is the meaning of these adjectives as thus employed? The question admits of a perfectly clear answer, as follows: Consider the expression

$$u_1x_1 + u_2x_2 + u_3x_3 + \cdots + u_nx_n + 1.$$

Denote it by the symbol $E(u, x)$. Observe that the u's enter the expression just as the x's enter it, it being quite indifferent, owing to the law of commutation here assumed, whether a given u comes before its x or after it. $E(u, x)$ is, on this account, said to be symmetric as to the u's and the x's. Next consider the equation, $E(u, x) = 0$. It, too, for the same reason, is symmetric in the same respect. The x's may be taken to be the coordinates of a conceptual point in a conceptual space S_n of n dimensions. Then the equation, imposing one condition on the variability of the x's, will represent or define in S_n an $(n-1)$-fold infinitude of points constituting in S_n a conceptual space S_{n-1} of $n-1$ dimensions. It is plain that to two different sets of values of the u's there will correspond two distinct spaces of the type S_{n-1}. Hence in S_n the u's may be taken to be the coordinates of the element S_{n-1} just as the x's were taken to be the coordinates of the point. The two conceptual interpretations, one of the x's and one of the u's, are naturally described as symmetric interpretations. Note carefully that, if the u's be held fast and the x's be allowed to vary subject to the condition, $E(u, x) = 0$, this equation represents some S_{n-1} as an ensemble of the points contained in it; and that, if the x's be held fast and the u's be allowed to vary subject to the condition, $E(u, x) = 0$, the equation represents a point as an ensemble of the spaces S_{n-1} containing it. Naturally these two interpretations, one for the u's constant and the x's variable, the other for the x's constant and the u's variable, of one and the same symmetric equation, $E(u, x) = 0$ or $E(x, u) = 0$, are to be themselves characterized as symmetric. The same is true of the following pairs of interpretations. If $E_1(u, x) = 0$ be a definite space S'_{n-1}, and $E_2(u, x) = 0$ represent another definite space S''_{n-1}, then the equation, $E_1(u, x) + \lambda E_2(u, x) = 0$, as λ varies, will represent, one after another, the single infinitude of spaces S_{n-1} that have in common the intersection, an S_{n-2}, of S'_{n-1} and S''_{n-1}; whilst, if the first two equations be interpreted to represent two points P' and P'', the third will, as λ varies, represent, one after another, the single infinitude of points on the line determined by P' and P''. Once more, according as $E_1(u, x) = 0$, $E_2(u, x) = 0$, and $E_3(u, x) = 0$ be construed to represent three independent spaces S'_{n-1}, S''_{n-1}, and S'''_{n-1} or three independent points P', P'', and P''', the equation, $E_1 + \lambda E_2 + \mu E_3 = 0$, will, as λ and μ vary, represent, one after another, either the twofold infinitude of spaces S_{n-1} having in common the intersection, an S_{n-3}, of S', S'', and S''', or the twofold infinitude of points in the plane determined by P', P'', and P'''. And so on and so on. Let not the reader falter if as yet he fail to see the outcome, for this will presently be clear.

Let it be noted and borne in mind that the foregoing pairs of symmetric interpretations are conceptual interpretations, interpretations by, in, and for thought—thought-interpretations, as they may be conveniently called. Denote the two of an arbitrary pair of them by $T(u)$ and $T(x)$, according as the *u's* or the *x's* are constant. Now imagination attempts in *its* way to make interpretations that correspond to—I do not say copy—those made by thought. In other words, imagination seeks to behold in its peculiar kind of light objects that correspond to or match the objects beheld by thought in its own kind of light. And in this function imagination succeeds perfectly if n be not too large, and never completely fails, however large be the value of n. Denote the imagination interpretations corresponding to $T(u)$ and $T(x)$ by $I(u)$ and $I(x)$, respectively. Now suppose that $n=2$; then S_n is a plane. Then, if $T(u)$ is the interpretation of $E(u, x)=0$, $T(u)$ is the *concept* of a range of points, the ensemble of the points of a line, and $I(u)$ is the corresponding object in imagination's light, namely, the so-called "image" or "picture" of a range of points; whilst $T(x)$ is the concept of a pencil of lines, the totality of lines (of the plane) through a point, and $I(x)$ is the "image" of a pencil. Still keeping $n=2$, if $T(u)$ is the interpretation of the equation, $E_1(u, x) + \lambda E_2(u, x) = 0$, then $T(u)$ is the concept: a variable range of a pencil (of ranges) determined by the ranges S' and S'', and $I(u)$ is the corresponding thing beheld by the imagination; whilst $T(x)$ is the concept: a variable point of a range determined by the points P' and P'', and $I(x)$ is the corresponding object in the field of imagination.

In order to gain all needed facility in the matter, let us pursue the parallelism in the case that $n=3$. It will be observed that here, just as when $n=2$, the objects presented to imagination perfectly match their correspondents presented to thought, though an object of the one set is as radically different from its correlative in the other as the definition or concept of a type of harmony is different from the harmony itself to a deaf Beethoven. As n is now 3, S_n is ordinary space. If $T(u)$ and $T(x)$ be the thought interpretations of the equation, $E(u, x)=0$, then the former is the concept of a plane of points, the latter is the concept of a point (or bundle or sheaf) of planes, the totality of planes having a point in common; whilst $I(u)$ and $I(x)$ are respectively imagination's correlatives of the two concepts. If $T(u)$ and $T(x)$ are interpretations of the equation, $E_1(u, x) + \lambda E_2(u, x) = 0$, then the former is the concept of a variable plane of the axal pencil (of planes) determined by the planes S' and S'', *i. e.*, the total ensemble of planes having a range of points in common, and $T(x)$ is the symmetric of the other, namely, the concept of a variable point of the range (of points) determined by the points P' and P'', the total ensemble of points common to a pencil of planes; whilst $I(u)$ and $I(x)$ are the things representing those concepts in imagination. Finally, denoting by the *T's* the conceptual interpretations of the equation,

$$E_1(u, x) + \lambda E_2(u, x) + \mu E_3(u, x) = 0,$$

the *E's* being supposed independent, then $T(u)$ is the concept of a variable plane of the bundle (of planes) determined by the three planes S', S'', S''', and $T(x)$ is the concept of a variable point of the plane (of points) determined by the three points P', P'', P'''.

One more step, and we shall have reached the goal. Before taking it, it is essential to note the following matters.

A.—As n increases more and more beyond the value 3, neither $T(u)$ nor $T(x)$ fails in any degree whatever, but both $I(u)$ and $I(x)$ fail in part, though not completely.

B.—For every value of n, $T(u)$ and $T(x)$ are symmetric interpretations; and so, too, are $I(u)$ and $I(x)$, though these two are incomplete for n equal to or greater than 4.

C.—From the fact that the interpretations $T(u)$ and $T(x)$ are symmetric, does it follow that the faculty, thought, which produces them is symmetric or functions symmetrically? No, for there are degrees of symmetry, so that of a pair of symmetric thought interpretations the one might be, as an interpretation, less complete than the other, in which case the faculty, thought, producing them would have to be said to function asymmetrically. Is, then, thought asymmetric? No, it is symmetric. Why? Because the $T(u)$ and $T(x)$ of any pair of symmetric interpretations are *equally*, being absolutely, complete. Is the faculty imagination symmetric? No, it is asymmetric. Why? Is it because for some values of n the interpretations, $I(u)$ and $I(x)$, although symmetric, are incomplete? No, imagination is said to be asymmetric or to function asymmetrically because, for higher values of n, the symmetric interpretations, $I(u)$ and $I(x)$, are, as we shall presently see, *unequally* incomplete, failing, that is, in respect to completeness as interpretations, in unequal measures.

To show this inequality and therewith to establish the thesis of this paper, it suffices to take n equal to 4. S_n is now S_4. The $T(u)$ of the equation, $E(u, x) = 0$, is the concept of a space, S_3, of points, and $T(x)$ is the concept of a threefold infinitude, a hypersheaf, of such spaces—call them lineoids—the total ensemble, then, of lineoids that, in four-space, have one and but one point in common. Now a lineoid of points and a point (or hypersheaf) of lineoids are logically or analytically exactly alike, any system of postulates serving as basis for the geometry of either of them serves equally well for the other, and either of the concepts gives an absolutely perfect and complete interpretation of the equation in question. But now consider imagination's correspondents of $T(u)$ and $T(x)$, namely $I(u)$ and $I(x)$. The former should be an "image" of a lineoid. Now a lineoid, or an S_3, being naught but an ordinary space, such as that in which we reside, is fairly well, though not completely, disclosed in the light and realm of imagination. $I(x)$, on the other hand, should be the "image" of a hypersheaf of lineoids, a threefold infinitude of lineoids enveloping a single point. But this "image"—and here is the crux of the matter—imagination fails to produce with a degree of completeness even remotely approximating that of $I(u)$. Here, then, in the case of $I(u)$ and $I(x)$ we have a perfectly clear and unmistakable example of asymmetry in the functioning of imagination. And the region we are operating in abounds in similar examples, many of them being even more striking in character. For another example, let $T(u)$ and $T(x)$ refer to the equation, $E_1(u, x) + \lambda E_2(u, x) = 0$. Then $T(u)$ is the concept: a variable lineoid of a pencil of lineoids, single infinitude of lineoids, determined by the lineoids S' and S''; $T(x)$ is the concept of a variable point (vertex of a hypersheaf of lineoids) of the point range determined by the points P' and P''. Here again the thought interpretations are perfect and complete. As to the corresponding "images," $I(u)$ and $I(x)$, no one can fail to see that in respect of completeness or incompleteness as matches for $T(u)$ and $T(x)$, the I's are widely asunder. A yet more illuminating example is that wherein $T(u)$ is the concept of a variable lineoid of the hyperpencil of a twofold infinitude of lineoids enveloping a line of S_4, where the corresponding $T(x)$ is the concept of a variable point of a plane, and where $I(u)$ and $I(x)$ are to be the corresponding objects in the light of imagination. It is needless, however, further to multiply examples from four-space, much less from spaces of higher dimensionality. The case is clear. In respect to spatial interpretation the power or faculty of thought is symmetric and that of imagination is asymmetric. The former, as n increases, looks about in spaces of ever increasing dimensionality like a binocular being with its twofold vision unimpaired, its light is spread abroad equally everywhere; whilst the eyes of imagination not only fail as n mounts higher and higher, but fail in *unequal* measure. Its light is but partial and fragmentary, scattered and suspended here, there, and yonder, like shining clouds in a sky filled and illuminated by a milder and gentler radiance.

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THE THEISTIC READJUSTMENT OF IDEALISM

ONCE the "cardinal principle of idealism" has been accepted as the fundamental doctrine of our philosophy, there are, roughly speaking, four probable views as to the constitution of the universe. First, thinkers may be divided into monists or absolutists and pluralists in respect of their attitude toward the reality of individual selves. And, secondly, as regards their belief in the reality or illusory nature of time, each of these may be classified as dynamic or static, respectively. The majority of historical systems, in their ontological aspects, may then be roughly arranged as belonging to one or other of those four groups, although the epistemological groundwork and historic setting will vary widely in different cases. Similarly the questions of the temporal and eternal, and of the one and the many, have been the leading *causæ belli* of historic philosophy. Both problems were familiar to Plato, the doctrine of ideas, as developed in the earlier dialogues, representing an attempted solution of both in the light of a single systematic principle.

Now these problems again reappeared when the early apologists of the Christian church turned to Greek philosophy in the search for a rational confirmation of the dogmas of their faith. Those dogmas had sprung from a Hebraic and instinctive source, and the simple conviction of the nearness, the omnipresence, of God, which, as Renan has shown, formed the content of primitive Christianity, now came under the influence of the speculative tendencies of the Hellenic mind. Accordingly, various new readjustments of speculative philosophy occurred, always conditioned by the reality of a divine being and an order of things other than the kingdom of this world. These readjustments, however, always took place in terms of the fundamental questions we mentioned above, and the dogma of the Trinity is an example of the result.

Such was the intellectual atmosphere prevalent throughout the Middle Ages, after which the revival of classical culture and the rise of the natural and mathematical sciences attracted attention once more to the temporal and dynamic, and opened afresh the controversy as to where finality might be found. From that time onward the fundamental problems of philosophy, from a strictly metaphysical point of view, have remained the same, though their solution